Combination: It means **Selection.** Order of things has no importance.
Permutation: It means **Selection and Arrangement**. Order of things has importance.

Q1) In how many ways can we select 4 men from 10 men?
Solution: C(10,4)

Q2) In how many ways can we select 4 boys and 5 girls out of 10 girls and 20 boys?
Solution: $C(10,5) \times C(20,4)$

Q3) In how many ways can we arrange 4 men out of 10 men?
Solution: $C(10,4) \times 4! = P(10,4)$

Formation of a committee from a given set of persons

Q4) In how many ways can 5 members form a committee out of 10 be selected such that
1) two particular members must be included.
2) two particular members must not be included.

Solution:
1) When two particular members are included then, we have to select 5 – 2 = 3 members out of 10 – 2 = 8
Required ways = $C(8,3) = 56$

2) When two particular members are not included then, we have to select 5 members out of 10 – 2 = 8
Required ways = $C(8,5) = 56$

Linear Arrangement: 'n' things can be arranged in a linear manner in $n!$. Linear arrangement refers to arrangement of people in a row or when people are seated on a bench etc.

Q5) In how many ways can we arrange 6 boys?
Solution: 6! ways.

Number of permutations of n objects out of which p are alike and are of one type, q are alike and are of second type, r are alike and are of third type $\frac{\boldsymbol{n}!}{\boldsymbol{p}!\boldsymbol{q}!\boldsymbol{r}!}$

Q6) Total number of permutations of 'INHALE'?
Solution: As there are 6 alphabets. So, total number of permutations are 6!

Q7) Total number of permutations of 'MISSISSIPPI'?
Solution: $\frac{11!}{4!4!2!}$

Q8) In how many ways can we arrange 6 boys and 6 girls such that all the girls are together?

Solution: As girls are always together so they are considered as a group. So there are 7 things to be arranged (6 boys, group of girls). They can be arranged in 7! ways.
Now internal arrangement of girls can be done in 6! ways.
Total ways = $7! \times 6!$ ways

Q9) In how many ways can 6 physics, 5 chemistry and 4 math books be arranged, if all the books of each type are to be kept together?

Solution: As books of the same subject are together so, there are 3 groups. These three groups can be arranged in 3! ways.
Now, internal arrangement of 6 physics books are done in 6! ways, of 5 chemistry books in 5! ways and of 4 maths books in 4! ways.
Total ways = $3! \times 6! \times 5! \times 4!$ ways

Q10) Let us consider the word FAILURE?

1) Total number of arrangements?
Solution: As there are 7 alphabets. So, total number of arrangements are 7!

2) How many words start from F?
Solution: $1 \times 6! = 6!$ ways

3) How many words end with E?
Solution: $6! \times 1 = 6!$ ways

4) How many words start from F and end with E?
Solution: $1 \times 5! \times 1 = 5!$ ways

5) In how many words vowels are always together?

Solution: Vowels: AIEU Consonants: FRL
As vowels are always together so 'AIEU' is considered as a group. So there are 4 things to be arranged (F, R, L, group of vowels). These 4 things can be arranged in 4! ways.
Now internal arrangement of vowels can be done in 4! ways.
Total ways = $4! \times 4!$ ways

6) In how many words all vowels are not together?

Solution: Required ways = Total ways – Ways in which all vowels are together
Total ways = 7!
Ways in which all vowels are together = $4! \times 4!$
Required ways = $7! - 4! \times 4!$ ways

7) In how many words no two vowels are together?

Solution: No two vowels are together means that consonants are between 2 vowels.
x F x L x R x
Consonants F, L, R are arranged in 3! ways. Vowels have 4 places (marked by 'x') to be occupied. In order to place 4 vowels all the 4 places need to be selected. This can be done in $C(4,4)$ ways. Now the vowels are placed in 4! ways.
So, total ways = $3! \times C(4,4) \times 4! = 3! \times 4!$ ways

8) In how many words vowels occupy odd positions.

Solution: Out of total 7 positions, vowels occupy odd positions. In order to place 4 vowels all the 4 odd places need to be selected. This can be done in $C(4,4)$ ways. Now the vowels are placed in 4! ways. Now, 3 positions are vacant and 3 consonants can be placed in those 3 positions in 3! ways.
So, total ways = $C(4,4) \times 4! \times 3! = 4! \times 3!$ ways

9) In how many ways no two consonants are together.

Solution: No two consonants are together means that consonants are separated by vowels.
x A x I x E x U x
Consonants A, I, O, U are arranged in 4! ways. Consonants have 5 places (marked by 'x') to be occupied. In order to place 3 consonants 3 places need to be selected out of 5 places marked by 'x'. This can be done in $C(5,3)$ ways. Now the consonants are placed in 3! ways.
So, total ways = $4! \times C(5,3) \times 3!$ ways

10) In how many words consonants occupy odd positions.

Solution: Out of total 7 positions, consonants occupy odd positions. In order to place 3 consonants (F, L, R) we require 3 odd positions out of 4 odd positions. This can be done in $C(4,3)$ ways. Now the consonants are placed in 3! ways. Now, 4 positions are vacant and 4 vowels can be placed in 4! ways.
So, total ways = $C(4,3) \times 3! \times 4!$ ways

11) No two consonants are together means that consonants are separated by vowels.

Solution: x A x I x E x U x
Consonants A, I, O, U are arranged in 4! ways. Consonants have 5 places (marked by 'x') to be occupied. In order to place 3 consonants 3 places need to be selected out of 5 places marked by 'x'. This can be done in $C(5,3)$ ways. Now the consonants are placed in 3! ways.
So, total ways = $4! \times C(5,3) \times 3!$ ways

Q11) Let us consider the word INTERNATIONAL?

1) Total number of arrangements?

Solution: $\frac{13!}{(2!)^3 3!}$

2) In how many words vowels are always together?
Solution: $\frac{8!6!}{(2!)^3 3!}$

3) In how many words consonants are always together?
Solution: $\frac{7!7!}{(2!)^3 3!}$

Q12) In how many words can be formed using the letters of the word 'SESSIONS' such that all 'S' are in alternate positions?

Solution: S x S x S x S x
S are arranged in 1 way. Now, the remaining 4 alphabets will occupy the remaining 4 places (marked by 'x') in 4! ways.
Another arrangement that is possible is 'x S x S x S x S'
S are arranged in 1 way. Now, the remaining 4 alphabets will occupy the remaining 4 places (marked by 'x') in 4! ways.
So, total ways = $4! + 4! = 2 \times 4!$ ways

Q13) In how many ways can we arrange 6 boys and 6 girls such that no two girls are together?

Solution: x B x B x B x B x B x B x
6 boys are arranged in 6! ways. Now, 6 girls will occupy 6 places out of the remaining 7 places (marked by 'x'). 6 places are chosen out of the remaining 7 places in $C(7,6)$ ways. 6 girls are arranged on these 6 places in 6! ways. So, total ways = $6! \times C(7,6) \times 6!$ ways

Circular Arrangement: 'n' objects can be arranged in a circular manner in $(n-1)!$ ways.

Q14) In how many ways can we arrange 6 boys in a circle?
Solution: 5! ways.

Q15) Find the number of ways, in which 10 boys can form a ring?
Solution: 9! ways.
In case, of necklace/garland/beads. 'n' objects can be arranged in a circular manner in $\frac{(n-1)!}{2}$ ways.

Q16) Find the number of ways in which 13 different beads can be arranged to form a necklace.
Solution: $\frac{(13-1)!}{2} = \frac{12!}{2}$

Concept of Handshakes

Let there be ‘n’ persons in a hall. If every person shakes his hand with every other person only once, then the total number of handshakes = $C(n, 2)$
If in place of handshakes each person gives a gift to another person, then formula changes to $2 \times C(n, 2)$

If there are n non collinear points in a plane, then
1) Number of straight lines formed = $C(n, 2)$
2) Number of triangles formed = $C(n, 3)$
3) Number of quadrilaterals formed = $C(n, 4)$

Q17) In a plane there are 16 non- collinear points. Find the number of straight lines formed.
Solution: Number of straight lines formed = $C(16,2) = 120$

If there are n points in a plane out of which m are collinear, then
1) Number of straight lines formed = $C(n, 2) - C(m, 2) + 1$
2) Number of triangles formed = $C(n, 3) - C(m, 3)$

Q18) If there are 11 points in a plane out of which 5 are collinear. Find the number of triangles made by these points.
Solution: Number of triangles formed = $C(11,3) - C(5,3) = 165 - 10 = 155$

Q19) In how many ways can we arrange 4 boys and 4 girls in a circle such that no two girls are together?

Solution: 4 boys are arranged in 3! ways. Now, there are 4 places in between 4 boys. 4 girls will occupy 4 places. 4 places are chosen out of 4 places in $C(4,4)$ ways. 4 girls are arranged on these 4 places in 4! ways. So, total ways = $3! \times C(4,4) \times 4! = 3! \times 4!$ ways

Q20) In how many ways can we arrange 6 boys and 4 girls in a circle such that no two girls are together?
Solution:
6 boys are arranged in 5! ways. Now, there are 6 places in between 6 boys. 4 girls will occupy 4 places. 4 places are chosen out of 6 places in $C(6,4)$ ways. 4 girls are arranged on these 4 places in 4! ways. So, total ways = $5! \times C(6,4) \times 4!$ ways

Q21) In how many ways can we arrange 7 silver oranges.
Solution: As, all the silver oranges are alike (identical) so, 1 way.

Q22) A class photograph is to be taken. The front row consists of 5 girls who are sitting. 18 boys are standing behind. The two corner positions are reserved for the 2 tallest boys. In how many ways can the students be arranged?

Solution: 5 girls are arranged in 5! ways. Now, the 2 tallest boys are arranged on the corner positions in 2! ways and the remaining 16 boys in 16! ways. So, total ways = $5! \times 16! \times 2!$ ways.

Q23) A gentleman has 6 friends to invite. In how many ways, can he send invitation cards to them, if he has three servants to carry the cards?

Solution: Invitation card can be sent to each of the six friends by any one of the three servants in 3 ways. Required ways = $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

Formation of numbers with given digits:

In these type of questions, it is asked to form numbers with some different digits. These digits can be used with repetition or without repetitions.

Q24) How many numbers of four digits can be formed with the digits 1, 2, 3, 4?
1) If repetition of digits is not allowed.
Solution: Total numbers formed = $4 \times 3 \times 2 \times 1 = 24$

2) If repetition of digits is allowed.
Solution: Total numbers formed = $4 \times 4 \times 4 \times 4 = 4^4$

Q25) How many numbers of four digits can be formed with the digits 1, 2, 3, 4, 5?
1) If repetition of digits is not allowed.
Solution: Total numbers formed = $5 \times 4 \times 3 \times 2 = 120$

2) If repetition of digits is allowed.
Solution: Total numbers formed = $5 \times 5 \times 5 \times 5 = 5^4$

Q26) How many numbers of four digits can be formed with the digits 1, 2, 3, 4?
1) If repetition of digits is not allowed.
Solution: Total numbers formed = $4 \times 3 \times 2 \times 1 = 24$

2) If repetition of digits is allowed.
Solution: Total numbers formed = $4 \times 4 \times 4 \times 4 = 4^4$

Q27) How many numbers of four digits can be formed with the digits 1, 2, 3, 4, 5?
1) If repetition of digits is not allowed.
Solution: Total numbers formed = $5 \times 4 \times 3 \times 2 = 120$

2) If repetition of digits is allowed.
Solution: Total numbers formed = $5 \times 5 \times 5 \times 5 = 5^4$

Q28) How many numbers of four digits can be formed with the digits 0, 1, 2, 3?
1) If repetition of digits is not allowed.
Solution: Total numbers formed = $3 \times 3 \times 2 \times 1 = 18$

2) If repetition of digits is allowed.
Solution: Total numbers formed = $3 \times 4 \times 4 \times 4 = 192$

Q29) How many numbers of four digits greater than 999 and not greater than 4000 can be formed with the digits 0, 1, 2, 3, 4? (If repetition of digits is allowed)

Solution: Numbers formed between 1000 and 3999 = $3 \times 5 \times 5 \times 5 = 375$
Total numbers = 375 + 1 (for the number 4000) = 376