Basic Concept, Important Formulas and Shortcuts: Surds and Indices

INDICES / EXPONENTS

The expression $a^n = a \times a \times a \times \ldots\ldots$n times where 'a' is the base and 'n' is the exponent.

Example: $a^3 = a \times a \times a$, $a^5 = a \times a \times a \times a \times a$

1) $\boldsymbol{a^m \times a^n = a^{m+n}}$

Example 1: $2^3 \times 2^4 = 2^{3+4} = 2^7$

Example 2: $2^{0.5} \times 2^3 = 2^{0.5+3} = 2^{3.5}$

Example 3: $2^{0.7} \times 2^{3.1} = 2^{0.7+3.1} = 2^{3.8}$

2) $\boldsymbol{\frac{a^m}{a^n} = a^{m-n}}$ **, where a ≠ 0**

Example 4: $\frac{3^7}{3^2} = 3^{7-2} = 3^5$

Example 5: $\frac{3^7}{3^{4.5}} = 3^{7-4.5} = 3^{2.5}$

Example 6: $\frac{3^{4.1}}{3^{1.3}} = 3^{4.1-1.3} = 3^{2.8}$

3) $\boldsymbol{(ab)^m = a^m b^m}$

Example 7: $(10)^3 = (2\times5)^3 = 2^3 \times 5^3$

4) $\boldsymbol{(a^m)^n = a^{mn}}$

Example 8: $(2^4)^3 = 2^{12}$

5) $\boldsymbol{a^0 = 1}$ **(For any non zero real number 'a')**

6) $\boldsymbol{a^{-m} = \frac{1}{a^m}}$ **(For any non zero real number 'a' and any integer 'm')**

Example 9: $3^{-4} = \frac{1}{3^4}$

7) $\boldsymbol{a^{1/n} = \sqrt[n]{a}}$

Example 10: $3^{1/4} = \sqrt[4]{3}$

Example 11: $3^{1/5} = \sqrt[5]{3}$

8) $\boldsymbol{a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m}$

Example 12: $5^{3/4} = \sqrt[4]{5^3}$

Example 13: $4^{2/3} = \sqrt[3]{4^2}$

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9) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ **, where b ≠ 0**

10) If $a^m = a^n$ **then m = n**

Q1) Simplify each of the following

1) $2^3 \times 2^4$ **2)** $\frac{3^6}{3^2}$ **3)** 4^0 **4)** $(-2.5)^0$

Solution:

1) $2^3 \times 2^4 = 2^7$ 2) $\frac{3^6}{3^2} = 3^4$ 3) $4^0 = 1$ 4) $(-2.5)^0 = 1$

Q2) Write each of the following with positive exponents

1) 2^{-5} **2)** $\frac{1}{(0.37)^{-5}}$ **3)** $\frac{x^{-5}}{y^{-2}}$ **4)** $\frac{y^{-5}}{x^3}$

Solution:

1) $2^{-5} = \frac{1}{2^5}$ 2) $\frac{1}{(0.37)^{-5}} = (0.37)^5$

3) $\frac{x^{-5}}{y^{-2}} = \frac{y^2}{x^5}$ 4) $\frac{y^{-5}}{x^3} = \frac{1}{x^3 y^5}$

Q3) Simplify each of the following:

1) $\frac{36x^{11}}{9x^7}$ **2)** $(2k^{-3})^4$ **3)** $(3pq^{-2})^2(2p^{-1}q)^3$

Solution:

1) $\frac{36x^{11}}{9x^7} = 4x^4$ 2) $(2k^{-3})^4 = 2^4 k^{-12} = \frac{2^4}{k^{12}}$

3) $(3pq^{-2})^2(2p^{-1}q)^3 = (9p^2q^{-4})(8p^{-3}q^3) = 72p^{-1}q^{-1} = \frac{72}{pq}$

Q4) Find the value of the following expression $3^{a+x}.3^{a-x}.3^{x-2a}$

1) 3^{-2a} **2)** 3^a **3)** 3^x **4) 1**

Solution: $3^{a+x}.3^{a-x}.3^{x-2a} = 3^{a+x+a-x+x-2a} = 3^x$

Q5) Find the value of x **such that** $2^{2x-3} = 2^{3x+2}$

1) 1 **2)** -5 **3) 5** **4) None of these**

Solution: $2x - 3 = 3x + 2$

On solving, $x = -5$

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Q6) $16^{7.5} \div 8^{3.5} \div 2^{7.5} = ?$ **SBI 2005**

1) 8^4 **2)** 16^4 **3)** 2^{15} **4)** 2^{27}

Solution: $16^{7.5} \div 8^{3.5} \div 2^{7.5} = 2^{30} \div 2^{10.5} \div 2^{7.5}$

$= 2^{30-10.5} \div 2^{7.5}$ $(a^m \div a^n = a^{m-n})$

$= 2^{19.5} \div 2^{7.5} = 2^{12} = 2^{3\times4} = (2^3)^4$ $\{a^{mn} = (a^m)^n\}$

$= 8^4$

Q7) Evaluate $a^p(2a)^q = ?$

1) $2a^{p+q}$ **2)** $(2a)^{p+q}$ **3)** $2^q a^{p+q}$ **4)** $2^q a^{pq}$

Solution: $a^p(2a)^q = a^p 2^q a^q = 2^q a^{p+q}$

Q8) $4^{15} \times 9^{-7} \times 18^{11} = 4^p \times 9^q$**. Find the values of** q, p**.**

1) $41, 36$ **2)** $4, \frac{41}{2}$ **3)** $\frac{41}{2}, 4$ **4)** $36, 41$

Solution: $4^{15} \times 9^{-7} \times 18^{11} = 2^{30} \times 3^{-14} \times 2^{11} \times 3^{22} = 2^{41} \times 3^8 = 4^{20.5} \times 9^4$

Q9) If $a^3 + 3^{a/2} = 11$**, then find the value of** a**.**

1) 1 **2) 4** **3) 2** **4) None of these**

Solution: Check options, $a = 2$

Q10) $25^{3.75} \times 49^{2.85} \times 125^{2.6} \times 343^{3.2} = 35^?$ **Bank PO 2014**

1) 16.3 **2) 16.1** **3) 15.3** **4) 15.7**

Solution: $25^{3.75} \times 49^{2.85} \times 125^{2.6} \times 343^{3.2} = 5^{7.5} \times 7^{5.7} \times 5^{7.8} \times 7^{9.6}$

$= 5^{15.3} \times 7^{15.3} = 35^{15.3}$

Q11) $100! = k \times 2^p$**, where** p **is the maximum exponent of 2. Then** k **satisfies which of the following properties.**

1) k **is an even integer** **2)** k **is an odd integer**

3) k **is a prime number** **4) None of these**

Solution: As p is the maximum exponent of 2. This means that k is an odd integer.

Q12) Find the sum of all the possible values of x **such that** $3^{|3x+5|} = 81$

1) $\frac{-1}{3}$ **2)** $\frac{-10}{3}$ **3)** -3 **4) None of these**

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Solution: $|3x + 5| = 4$

$$3x + 5 = \pm 4 \text{ {If } } |a| = k \text{ then } a = \pm k\}$$

On solving, $x = -3 \; or \; \frac{-1}{3}$

Sum = $-3 + \left(\frac{-1}{3}\right) = -\frac{10}{3}$

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