

Maxima Minima Questions from Previous Year CAT Papers 2008-2017

Homogeneous condition: It is a condition in which on interchanging the variables, the condition remains the same.

Homogeneous expression: It is an expression in which on interchanging the variables, the expression remains the same.

NOTE: Whenever we are given a homogeneous condition and we have to find the maximum/minimum value of a homogeneous expression, then all the variables are equal.

Q1) If $x + y = 10$ then maximum value of xy ?

Solution: In this question, both the condition and the expression are homogeneous in nature (If x, y are interchanged then, the condition and the expression remain the same) so, variables x, y are equal i.e. $x = y = 5$
Maximum value of $xy = 5 \times 5 = 25$

Q2) If $xy = 64$ then minimum value of $x + y$?

Solution: In this question, both the condition and the expression are homogeneous in nature (If x, y are interchanged then, the condition and the expression remain the same) so, variables x, y are equal i.e. $x = y = 8$
Minimum value of $x + y = 8 + 8 = 16$

Q3) If $a + b + c = 1$ then maximum value of $(1 - a)(1 - b)(1 - c)$? CAT 2016 Type

Solution: In this question, both the condition and the expression are homogeneous in nature (If a, b, c are interchanged then, the condition and the expression remain the same) so, variables a, b, c are equal i.e. $a = b = c = \frac{1}{3}$
Maximum value of $(1 - a)(1 - b)(1 - c) = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$

Q4) If $a + b + c + d = 1$ for non negative a, b, c then maximum value of $(1 + a)(1 + b)(1 + c)(1 + d)$? CAT 2017 Type

Solution: In this question, both the condition and the expression are homogeneous in nature (If a, b, c, d are interchanged then, the condition and the expression remain the same) so, variables a, b, c, d are equal i.e. $a = b = c = d = \frac{1}{4}$
Maximum value of $(1 + a)(1 + b)(1 + c)(1 + d) = \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} = \frac{625}{256}$

Q5) If $x + y + z + w = 1$ then minimum value of

$\left(x + \frac{1}{x}\right)^2 + \left(y + \frac{1}{y}\right)^2 + \left(z + \frac{1}{z}\right)^2 + \left(w + \frac{1}{w}\right)^2$ **Previous CAT**

Maxima Minima Questions from Previous Year CAT Papers 2008-2017

Solution: In this question, both the condition and the expression are homogeneous in nature (If x, y, z, w are interchanged then, the condition and the expression remain the same) i.e. variables x, y, z, w are equal. So, $x = y = z = w = \frac{1}{4}$

$$\begin{aligned} \text{Minimum value of } & \left(x + \frac{1}{x}\right)^2 + \left(y + \frac{1}{y}\right)^2 + \left(z + \frac{1}{z}\right)^2 + \left(w + \frac{1}{w}\right)^2 \\ & = \left(\frac{17}{4}\right)^2 + \left(\frac{17}{4}\right)^2 + \left(\frac{17}{4}\right)^2 + \left(\frac{17}{4}\right)^2 = 4 \times \left(\frac{17}{4}\right)^2 = \frac{289}{4} \end{aligned}$$

Q6) If $a + b + c + d = 2$, then what is the maximum value of $(1 + a + b)(1 + c + d)$?

Previous XAT

Solution: Put $a + b = P, c + d = Q$ such that the given condition is updated as $P + Q = 2$ (if P, Q are interchanged, still the condition remains the same i.e. homogeneous condition)

$$(1 + a + b)(1 + c + d) = (1 + P)(1 + Q)$$

In this question, both the condition and the expression are homogeneous in nature so, variables P, Q are equal. So, $P = Q = 1$

$$\text{Maximum value of } (1 + P)(1 + Q) = 4$$

Q7) If $a, b, c,$ and d are positive numbers such that $abcd = 1$, what is the minimum value of $(1 + a)(1 + b)(1 + c)(1 + d)$?

CAT 2015 Type

Solution: In this question, both the condition and the expression are homogeneous in nature (If a, b, c, d are interchanged then, the condition and the expression remain the same) i.e. variables a, b, c, d are equal. So, $a = b = c = d = 1$

$$\text{Minimum value of } (1 + a)(1 + b)(1 + c)(1 + d) = 16$$

Q8) If $a > 0, b > 0, a + b = 1$, what is the minimum value of $\left(a + \frac{1}{a}\right)\left(b + \frac{1}{b}\right)$?

Previous CAT

Solution: In this question, both the condition and the expression are homogeneous in nature (If a, b are interchanged then, the condition and the expression remain the same) i.e. variables a, b are equal. So, $a = b = \frac{1}{2}$

$$\text{Minimum value of } \left(a + \frac{1}{a}\right)\left(b + \frac{1}{b}\right) = \frac{25}{4}$$