

If A and B are in the ratio $m:n$ in a mixture so, whenever some quantity of the mixture is withdrawn, the withdrawn quantity will have A and B in the ratio $m:n$. If x litres is withdrawn then, in these x litres $A = \frac{mx}{m+n}$ $B = \frac{nx}{m+n}$

Repeated Dilution of a Mixture:

If a container initially contains V unit of liquid and x unit of liquid is taken out and it is filled with x unit of another liquid, then after n operations, the final quantity of the original liquid in the container is given as $V \left(1 - \frac{x}{V}\right)^n$

Q1) Two vessels contain equal quantity of mixtures of milk and water in the ratio 9:5 and 4:3 respectively. Both the mixtures are now mixed thoroughly. Find the ratio of milk to water in the new mixture so obtained.

- 1) 17:11 2) 11:17 3) 8:13 4) 13:8

Solution:

Vessel 1: 9:5 Vessel 2: 4:3

Volume of vessels = LCM of (9+5, 4+3) = 14 litres.

For Vessel 1: Milk = 9 litres, Water = 5 litres

For Vessel 2: Milk = 8 litres, Water = 6 litres

On mixing, Milk = 9 + 8 = 17 litres, Water = 5 + 6 = 11 litres

Ratio of milk to water = 17:11

Q2) The contents of two vessels containing water and milk are in the ratio 2:3 and 4:5 are mixed in the ratio 1:2. The resulting mixture will have water and milk in the ratio.

- 1) 77:58 2) 58:77 3) 68:77 4) 77:68

Solution:

Vessel 1: 2:3 Vessel 2: 4:5

LCM of (2+3, 4+5) = 45

As contents of two vessels are mixed in the ratio 1:2

For vessel 1: Volume = 45 litres Milk = $\frac{3}{5} \times 45 = 27$ litres, Water = 18 litres

For vessel 2: Volume = 90 litres Milk = $\frac{5}{9} \times 90 = 50$ litres, Water = 40 litres

On mixing, Milk = 27 + 50 = 77 litres, Water = 18 + 40 = 58 litres

Ratio of water to milk = 58:77

Q3) A and B are two alloys of gold and copper prepared by mixing metals in proportions 7:2 and 7:11 respectively. If equal quantities of alloys are melted to form a third alloy C, the proportion of gold and copper in C will be

- 1) 5:9 2) 5: 7 3) 7:5 4) 9:5

Solution:

Alloy 1: 7:2 Alloy 2: 7:11

Quantity of alloys = LCM of (7+2, 7+11) = 18 Kg.

For alloy 1: gold = 14 Kg, copper = 4 Kg

For alloy 2: gold = 7 Kg, copper = 11 Kg

On mixing, gold = 14 + 7 = 21 Kg, copper = 4 + 11 = 15 Kg

Ratio of gold to copper = 21:15 = 7:5

Q4) In 28 litres mixture of milk and water the ratio of milk and water is 5:2. How much water should be added in the mixture so that the ratio of milk to water becomes 2:5?

- 1) 42 litres 2) 32 litres 3) 24 litres 4) 39 litres

Solution:

Milk = $\frac{5}{7} \times 28 = 20$ litres, Water = 8 litres

Let x litres of water be added to the mixture so, $\frac{20}{8+x} = \frac{2}{5}$

On solving, x = 42 litres

Q5) A mixture contains milk and water in the ratio of 4:3. On adding 2 litres of water, the ratio of milk to water becomes 8:7. Find the quantity of the final mixture.

- 1) 16 litres 2) 12 litres 3) 28 litres 4) 30 litres

Solution:

Milk = 4x, Water = 3x

$$\frac{4x}{3x+2} = \frac{8}{7} \text{ i.e. } x = 4$$

Quantity of the final mixture = 4x + 3x + 2 = 7x + 2 = 30 litres.

Q6) A container contains 40 litres of milk. From this container, 4 litres of milk was taken out and replaced by water. This process was further repeated two times. How much milk is now there in the container?

- 1) 32.4 litres 2) 29.16 litres 3) 36 litres 4) Data Insufficient

Solution:

$$\text{Quantity of milk left} = 40 \left(1 - \frac{4}{40}\right)^3 = 29.16 \text{ litres}$$

Q7) A mixture contains milk and water in the ratio of 4:3. On adding 6 litres of water, the ratio of milk to water becomes 8:7. Find the quantity of the final mixture.

- 1) 168 litres 2) 90 litres 3) 42 litres 4) 84 litres

Solution:

$$\text{Milk} = 4x, \text{ Water} = 3x$$

$$\frac{4x}{3x+6} = \frac{8}{7} \text{ i.e. } x = 12$$

$$\text{Quantity of the final mixture} = 4x + 3x + 6 = 7x + 6 = 90 \text{ litres.}$$

Q8) A bucket contains a mixture of two liquids A and B in the proportion 6:5. If 33 litres of the mixture is replaced by 33 litres of liquid B, then the ratio of the two liquids becomes 3:4. How much of the liquid A was there in the bucket.

- 1) 64 litres 2) 70 litres 3) 48 litres 4) 84 litres

Solution:

$$\text{Initial mixture: } A = 6x, B = 5x$$

In 33 litres of the mixture that is replaced

$$A = \frac{6}{11} \times 33 = 18 \text{ litres } B = 15 \text{ litres}$$

Liquid	Initial Mixture	Final Mixture	
		After 33 litres of the mixture is replaced	After 33 litres of B is added
A	6x	6x - 18	6x - 18
B	5x	5x - 15	5x - 15 + 33 = 5x + 18

$$\frac{6x-18}{5x+18} = \frac{3}{4} \text{ i.e. } x = 14$$

$$\text{Liquid A} = 6(14) = 84 \text{ litres}$$

Q9) A vessel contains 24 litres of milk. 4 litres are withdrawn and replaced by water. The process is repeated a second time. Find the ratio of milk to water in the resulting mixture?

- 1) 25:36 2) 36:11 3) 11:25 4) 25:11

Solution:

Volume of milk left = Initial volume of milk $\left(1 - \frac{x}{V}\right)^n$

Initial volume of milk = 24 litres = V (Volume of the vessel)

x = 4 litres; n = 2 times

Volume of milk left = $24\left(1 - \frac{4}{24}\right)^2 = \frac{50}{3}$ litres

Ratio of milk to water in the resulting mixture = $\frac{\frac{50}{3}}{24 - \frac{50}{3}} = \frac{50}{22} = \frac{25}{11}$

Q10) A vessel contains liquid A and B in the ratio 3:1. If 8 litres of the mixture is replaced by 8 litres of liquid B, then the ratio of the two liquids becomes 1:3. What quantity does the vessel hold?

- 1) 12 litres 2) 14 litres 3) 16 litres 4) 10 litres

Solution:

Initial mixture: A = 3x, B = x

In 8 litres of the mixture that is replaced

$A = \frac{3}{4} \times 8 = 6$ litres B = 2 litres

Liquid	Initial Mixture	Final Mixture	
		After 33 litres of the mixture is replaced	After 33 litres of B is added
A	3x	3x - 6	3x - 6
B	x	x - 2	x - 2 + 8 = x + 6

$\frac{3x-6}{x+6} = \frac{1}{3}$ i.e. x = 3

Quantity that the vessel holds = 3x + x = 4x = 4(3) = 12 litres

Q11) From a cask of wine, containing 64 litres, 8 litres are drawn out and the cask is filled with water. If the same process is repeated a second, then a third time, what will be the proportion of wine to water in the resulting mixture?

- 1) 343:169 2) 343:512 3) 169:343 4) 512:343

Solution:

Volume of wine left = Initial volume of wine $\left(1 - \frac{x}{V}\right)^n$

Initial volume of wine = 64 litres = V (Volume of the vessel)

x = 8 litres; n = 3 times

Volume of wine left = $64\left(1 - \frac{8}{64}\right)^3 = \frac{343}{8}$ litres

Ratio of wine to water in the resulting mixture = $\frac{\frac{343}{8}}{64 - \frac{343}{8}} = \frac{343}{169}$

Q12) 8 litres are drawn off from a vessel full of water and substituted by pure milk. Again 8 litres of the mixture are drawn off and substituted by pure milk. If the vessel now contains water and milk in the ratio 9:40, find the capacity of the vessel.

- 1) 14 litres 2) 24 litres 3) 16 litres 4) 12 litres

Solution:

Let V be the volume of the vessel.

Volume of water left = Initial volume of water $\left(1 - \frac{x}{V}\right)^n$

$\frac{\text{Volume of water left}}{\text{Initial volume of water}} = \left(1 - \frac{x}{V}\right)^n$

x = 8 litres; n = 2 times

The vessel now contains water and milk in the ratio 9:40 i.e. Water: Total Mixture (Water + Milk) = 9:(9+40) = 9:49

$\frac{\text{Volume of water left}}{\text{Volume of water left} + \text{Volume of milk left}} = \left(1 - \frac{x}{V}\right)^n$

$$\frac{9}{49} = \left(1 - \frac{8}{V}\right)^2$$

$$1 - \frac{8}{V} = \frac{3}{7}$$

V = 14 litres

Q13) 10 litres are drawn off from a vessel full of wine. It is then filled up with water. Again 10 litres of the mixture are drawn off and the vessel is filled up with water. The ratio of the quantity of wine now left in the vessel is to that of the water in it as 144:25. Find the capacity of the vessel.

- 1) 135 litres 2) 120 litres 3) 130 litres 4) 140 litres

Solution:

Let V be the volume of the vessel.

Volume of wine left = Initial volume of wine $\left(1 - \frac{x}{V}\right)^n$

$$\frac{\text{Volume of wine left}}{\text{Initial volume of wine}} = \left(1 - \frac{x}{V}\right)^n$$

x = 10 litres; n = 2 times

The vessel now contains wine and water in the ratio 144:25 i.e. wine: Total Mixture (wine + water) = 144:(144+25) = 144:169

$$\frac{\text{Volume of wine left}}{\text{Volume of water left} + \text{Volume of wine left}} = \left(1 - \frac{x}{V}\right)^n$$

$$\frac{144}{169} = \left(1 - \frac{10}{V}\right)^2$$

$$1 - \frac{10}{V} = \frac{12}{13}$$

V = 130 litres