

Important Formulas from Ratio and Proportion

1. Ratio:

The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $a : b$. In the ratio $a : b$, we call a as the first term or **antecedent** and b , the second term or **consequent**.

Eg. The ratio $5 : 9$ represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Eg. $4 : 5 = 8 : 10 = 12 : 15$. Also, $4 : 6 = 2 : 3$.

2. Proportion:

The equality of two ratios is called proportion.

If $a : b = c : d$, we write $a : b :: c : d$ and we say that a, b, c, d are in proportion. Here a and d are called **extremes**, while b and c are called **mean terms**.

Product of means = Product of extremes.

Thus, $a : b :: c : d \Leftrightarrow (b \times c) = (a \times d)$.

3. Fourth Proportional:

If $a : b = c : d$, then d is called the fourth proportional to a, b, c .

Third Proportional:

$a : b = c : d$, then c is called the third proportion to a and b .

Mean Proportional:

Mean proportional between a and b is ab .

4. Comparison of Ratios:

We say that $(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$.

Compounded Ratio:

The compounded ratio of the ratios: $(a : b), (c : d), (e : f)$ is $(ace : bdf)$.

5. Duplicate Ratios:

Duplicate ratio of $(a : b)$ is $(a^2 : b^2)$.

Sub-duplicate ratio of $(a : b)$ is $(a : b)$.

▲ Triplicate ratio of $(a : b)$ is $(a^3 : b^3)$.

Sub-triplicate ratio of $(a : b)$ is $(a^{1/3} : b^{1/3})$.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. [componendo and dividendo]

6. Variations:

We say that x is directly proportional to y , if $x = ky$ for some constant k and we write, $x \propto y$.

We say that x is inversely proportional to y , if $xy = k$ for some constant k and

we write, $x \propto \frac{1}{y}$.

Practice Questions from Ratio and Proportion

1. A and B together have Rs. 1210. If $\frac{4}{15}$ of A's amount is equal to $\frac{2}{5}$ of B's amount, how much amount does B have?

- A. Rs. 460
 B. Rs. 484
 C. Rs. 550
 D. Rs. 664

Answer: Option B

Explanation:

$$\frac{4}{15} A = \frac{2}{5} B$$

$$\Rightarrow A = \left(\frac{2}{5} \times \frac{15}{4} \right) B$$

$$\Rightarrow A = \frac{3}{2} B$$

$$\Rightarrow \frac{A}{B} = \frac{3}{2}$$

$$\Rightarrow A : B = 3 : 2.$$

$$\therefore \text{B's share} = \text{Rs.} \left(1210 \times \frac{2}{5} \right) = \text{Rs.} 484.$$

2. Two numbers are respectively 20% and 50% more than a third number. The ratio of the two numbers is:

- A. 2 : 5
 B. 3 : 5
 C. 4 : 5
 D. 6 : 7

Answer: Option C

Explanation:

Let the third number be x .

$$\text{Then, first number} = 120\% \text{ of } x = \frac{120x}{100} = \frac{6x}{5}$$

$$\text{Second number} = 150\% \text{ of } x = \frac{150x}{100} = \frac{3x}{2}$$

$$\therefore \text{Ratio of first two numbers} = \left(\frac{6x}{5} : \frac{3x}{2} \right) = 12x : 15x = 4 : 5.$$

3. A sum of money is to be distributed among A, B, C, D in the proportion of 5 : 2 : 4 : 3. If C gets Rs. 1000 more than D, what is B's share?

- A. Rs. 500
- B. Rs. 1500
- C. Rs. 2000
- D. None of these

Answer: Option C

Explanation:

Let the shares of A, B, C and D be Rs. $5x$, Rs. $2x$, Rs. $4x$ and Rs. $3x$ respectively.

Then, $4x - 3x = 1000$

$\Rightarrow x = 1000$.

\therefore B's share = Rs. $2x = \text{Rs. } (2 \times 1000) = \text{Rs. } 2000$.

4. Seats for Mathematics, Physics and Biology in a school are in the ratio 5 : 7 : 8. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of increased seats?
- A. 2 : 3 : 4
 - B. 6 : 7 : 8
 - C. 6 : 8 : 9
 - D. None of these

Answer: Option A

Explanation:

Originally, let the number of seats for Mathematics, Physics and Biology be $5x$, $7x$ and $8x$ respectively.

Number of increased seats are (140% of $5x$), (150% of $7x$) and (175% of $8x$).

$\Rightarrow \left(\frac{140}{100} \times 5x \right)$, $\left(\frac{150}{100} \times 7x \right)$ and $\left(\frac{175}{100} \times 8x \right)$

$\Rightarrow 7x$, $\frac{21x}{2}$ and $14x$.

\therefore The required ratio = $7x : \frac{21x}{2} : 14x$

$\Rightarrow 14x : 21x : 28x$

$\Rightarrow 2 : 3 : 4$.

5. In a mixture 60 litres, the ratio of milk and water 2 : 1. If this ratio is to be 1 : 2, then the quantity of water to be further added is:
- A. 20 litres
 - B. 30 litres
 - C. 40 litres
 - D. 60 litres

Answer: Option D

Explanation:

$$\text{Quantity of milk} = \left(60 \times \frac{2}{3}\right) \text{ litres} = 40 \text{ litres.}$$

Quantity of water in it = (60 - 40) litres = 20 litres.

New ratio = 1 : 2

Let quantity of water to be added further be x litres.

$$\text{Then, milk : water} = \left(\frac{40}{20 + x}\right).$$

$$\text{Now, } \left(\frac{40}{20 + x}\right) = \frac{1}{2}$$

$$\Rightarrow 20 + x = 80$$

$$\Rightarrow x = 60.$$

∴ Quantity of water to be added = 60 litres.

6. The ratio of the number of boys and girls in a college is 7 : 8. If the percentage increase in the number of boys and girls be 20% and 10% respectively, what will be the new ratio?

- A. 8 : 9
- B. 17 : 18
- C. 21 : 22
- D. Cannot be determined

Answer: Option C

Explanation:

Originally, let the number of boys and girls in the college be $7x$ and $8x$ respectively. Their increased number is (120% of $7x$) and (110% of $8x$).

$$\Rightarrow \left(\frac{120}{100} \times 7x\right) \text{ and } \left(\frac{110}{100} \times 8x\right)$$

$$\Rightarrow \frac{42x}{5} \text{ and } \frac{44x}{5}$$

$$\therefore \text{The required ratio} = \left(\frac{42x}{5} : \frac{44x}{5}\right) = 21 : 22.$$

7. Salaries of Ravi and Sumit are in the ratio 2 : 3. If the salary of each is increased by Rs. 4000, the new ratio becomes 40 : 57. What is Sumit's salary?

- A. Rs. 17,000
- B. Rs. 20,000
- C. Rs. 25,500
- D. Rs. 38,000

Answer: Option D

Explanation:

Let the original salaries of Ravi and Sumit be Rs. $2x$ and Rs. $3x$ respectively.

Then, $\frac{2x + 4000}{3x + 4000} = \frac{40}{57}$

$$\Rightarrow 57(2x + 4000) = 40(3x + 4000)$$

$$\Rightarrow 6x = 68,000$$

$$\Rightarrow 3x = 34,000$$

Sumit's present salary = $(3x + 4000) = \text{Rs.}(34000 + 4000) = \text{Rs. } 38,000$.

8. If $0.75 : x :: 5 : 8$, then x is equal to:

A. 1.12

B. 1.2

C. 1.25

D. 1.30

Answer: Option B

Explanation:

$$(x \times 5) = (0.75 \times 8) \Rightarrow x = \left(\frac{6}{5}\right) = 1.20$$

9. The sum of three numbers is 98. If the ratio of the first to second is $2 : 3$ and that of the second to the third is $5 : 8$, then the second number is:

A. 20

B. 30

C. 48

D. 58

Answer: Option B

Explanation:

Let the three parts be A, B, C. Then,

$$A : B = 2 : 3 \text{ and } B : C = 5 : 8 = \left(5 \times \frac{3}{5}\right) : \left(8 \times \frac{3}{5}\right) = 3 : \frac{24}{5}$$

$$\Rightarrow A : B : C = 2 : 3 : \frac{24}{5} = 10 : 15 : 24$$

$$\Rightarrow B = \left(98 \times \frac{15}{49}\right) = 30.$$

10. If Rs. 782 be divided into three parts, proportional to $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$, then the first part is:

A. Rs. 182

B. Rs. 190

C. Rs. 196

D. Rs. 204

Answer: Option D

Explanation:

$$\text{Given ratio} = \frac{1}{2} : \frac{2}{3} : \frac{3}{4} = 6 : 8 : 9.$$

$$\therefore 1^{\text{st}} \text{ part} = \text{Rs.} \left(782 \times \frac{6}{23} \right) = \text{Rs.} 204$$

11. The salaries A, B, C are in the ratio 2 : 3 : 5. If the increments of 15%, 10% and 20% are allowed respectively in their salaries, then what will be new ratio of their salaries?

A. 3 : 3 : 10

B. 10 : 11 : 20

C. 23 : 33 : 60

D. Cannot be determined

Answer: Option C

Explanation:

Let A = 2k, B = 3k and C = 5k.

$$\text{A's new salary} = \frac{115}{100} \text{ of } 2k = \left(\frac{115}{100} \times 2k \right) = \frac{23k}{10}$$

$$\text{B's new salary} = \frac{110}{100} \text{ of } 3k = \left(\frac{110}{100} \times 3k \right) = \frac{33k}{10}$$

$$\text{C's new salary} = \frac{120}{100} \text{ of } 5k = \left(\frac{120}{100} \times 5k \right) = 6k$$

$$\therefore \text{New ratio} \left(\frac{23k}{10} : \frac{33k}{10} : 6k \right) = 23 : 33 : 60$$

12. If 40% of a number is equal to two-third of another number, what is the ratio of first number to the second number?

A. 2 : 5

B. 3 : 7

C. 5 : 3

D. 7 : 3

Answer: Option C

Explanation:

$$\text{Let } 40\% \text{ of } A = \frac{2}{3} B$$

$$\text{Then, } \frac{40A}{100} = \frac{2B}{3}$$

$$\Rightarrow \frac{2A}{5} = \frac{2B}{3}$$

$$\frac{A}{B} = \frac{5}{3}$$

$$\therefore A : B = 5 : 3.$$

13. The fourth proportional to 5, 8, 15 is:

- A. 18
- B. 24
- C. 19
- D. 20

Answer: Option B

Explanation:

Let the fourth proportional to 5, 8, 15 be x .

Then, $5 : 8 :: 15 : x$

$$\Rightarrow 5x = (8 \times 15)$$

$$x = \frac{(8 \times 15)}{5} = 24.$$

14. Two number are in the ratio 3 : 5. If 9 is subtracted from each, the new numbers are in the ratio 12 : 23. The smaller number is:

- A. 27
- B. 33
- C. 49
- D. 55

Answer: Option B

Explanation:

Let the numbers be $3x$ and $5x$.

$$\text{Then, } \frac{3x - 9}{5x - 9} = \frac{12}{23}$$

$$\Rightarrow 23(3x - 9) = 12(5x - 9)$$

$$\Rightarrow 9x = 99$$

$$\Rightarrow x = 11.$$

$$\therefore \text{The smaller number} = (3 \times 11) = 33.$$

15. In a bag, there are coins of 25 p, 10 p and 5 p in the ratio of 1 : 2 : 3. If there is Rs. 30 in all, how many 5 p coins are there?

- A. 50
- B. 100
- C. 150

D. 200

Answer: Option C

Explanation:

Let the number of 25 p, 10 p and 5 p coins be x , $2x$, $3x$ respectively.

Then, sum of their values = Rs. $\left(\frac{25x}{100} + \frac{10 \times 2x}{100} + \frac{5 \times 3x}{100}\right) = \text{Rs. } \frac{60x}{100}$

$$\therefore \frac{60x}{100} = 30 \quad \Leftrightarrow \quad x = \frac{30 \times 100}{60} = 50.$$

Hence, the number of 5 p coins = $(3 \times 50) = 150$.

www.prep4paper.com